

Frage:

Antwort:

Wichtige Limite:

$$1) \lim_{x \rightarrow 3} \frac{1}{(x-3)^2} = +\infty \quad \left(\frac{1}{0^+} \right)$$

$$2) \lim_{x \rightarrow \pm\infty} \frac{1}{(x-3)^2} = 0 \quad \left(\frac{1}{\infty} \right)$$

$$3) \lim_{x \rightarrow -1^{\pm}} \frac{-1}{x+1} = \mp \infty \quad \left(\frac{-1}{0^{\pm}} \right)$$

$$4) \lim_{x \rightarrow \pm 1^{\pm}} \frac{x-1}{x+1} = \mp \infty \quad \left(\frac{-2}{0^{\pm}} \right)$$

$$5) \lim_{x \rightarrow \pm\infty} \frac{x-1}{x+1} = 1 \quad \left(\frac{\infty}{\infty} \right)$$

$$6) \lim_{x \rightarrow \pm\infty} \frac{x^3 - 2x^2 + 1}{3x^2 + x} = \pm \infty \quad \left(\frac{\infty}{\infty} \sim \frac{x^3}{x^2} \right)$$

$$7) \lim_{x \rightarrow -1} \frac{x^2 - 2x - 3}{x^2 + 3x + 2} = \lim_{x \rightarrow -1} \frac{(x+1)(x-3)}{(x+1)(x+2)} = -4$$

$$8) \lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+1} + 1} = \frac{1}{2}$$

$$9) \lim_{x \rightarrow 0} \frac{\sin 3x}{x} = \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} \cdot 3 = 3$$

$$10) \lim_{x \rightarrow 0} \frac{\sin 3x}{x^3} = \lim_{x \rightarrow 0} \left(\frac{\sin 3x}{x} \cdot \frac{1}{x^2} \right) = +\infty \quad \left(3 \cdot \frac{1}{0^+} \right)$$

$$11) \lim_{x \rightarrow 0} \frac{\sin^2 3x}{x} = \lim_{x \rightarrow 0} \left(\frac{\sin 3x}{x} \cdot \sin 3x \right) = 0 \quad (3 \cdot 0)$$

$$12) \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{2x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{2x^2} = \lim_{x \rightarrow 0} \frac{1}{2} \left(\frac{\sin x}{x} \right)^2 = \frac{1}{2}$$

$$13) \lim_{x \rightarrow -3^+} \ln(x+3) = \lim_{y \rightarrow 0^+} \ln y = -\infty$$

$$14) \lim_{x \rightarrow +\infty} \ln \left(1 + \frac{1}{x} \right) = \lim_{y \rightarrow 1} \ln y = 0$$

$$15) \lim_{x \rightarrow +\infty} \frac{\sin x}{x} = 0 \quad \left(0 \leq \left| \frac{\sin x}{x} \right| \leq \frac{1}{x}, \lim_{x \rightarrow +\infty} \frac{1}{x} = 0 \right) \quad \checkmark$$