

ZT MAI 1 (14-15) - A

① $\lim_{n \rightarrow \infty} n \log\left(1 - \log \frac{2}{n}\right) = \infty \cdot 0^+ =$

$$= \lim_{n \rightarrow \infty} \frac{\log\left(1 - \log \frac{2}{n}\right)}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\log\left(1 - \log \frac{2}{n}\right) + \log \frac{2}{n} \left(-\frac{2}{n}\right) \cdot (-2)}{-\log \frac{2}{n} + \frac{2}{n} \cdot \frac{2}{n^2}} \cdot (-2)$$

$\underbrace{\hspace{10em}}_{\rightarrow 1} \quad \underbrace{\hspace{10em}}_{\rightarrow 1} \quad \text{VLSF } \frac{2}{n} \approx t$

$\left(-\log \frac{2}{n} = y\right) \xrightarrow[\text{VLSF}]{\text{H.V.}} = \lim_{y \rightarrow 0} \frac{\log(1+y)}{y} \cdot (-2) \stackrel{\text{H.V.}}{=} -2$

② $\sum_1^{\infty} (-1)^n \sin\left(\frac{n}{2+n^2}\right)$

a) abs. konvergenz:

$$\lim_{n \rightarrow \infty} \frac{\sin\left(\frac{n}{2+n^2}\right)}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\sin \frac{n}{2+n^2}}{\frac{n}{2+n^2}} \cdot \frac{n}{1+n^2} = 1, \quad \underbrace{\frac{n}{2+n^2} \rightarrow 1}_{\text{H.V. + VLSF}} \cdot \underbrace{\frac{n}{1+n^2} \rightarrow 1}_{\text{H.V. + VLSF}}$$

$\sum \frac{1}{n}$ divergiert \Rightarrow (L'Hôpital's rule) $\sum_1^{\infty} \sin\left(\frac{n}{1+n^2}\right)$ divergiert

b) absolut konvergenz: alleinstückig \Rightarrow $\left(0 < \frac{n}{2+n^2} \leq 1 < \frac{n}{2}\right) \Rightarrow \sin\left(\frac{n}{2+n^2}\right) > 0$

$$\lim_{n \rightarrow \infty} \sin\left(\frac{n}{2+n^2}\right) = \lim_{x \rightarrow 0} \sin\left(\frac{x}{2+x^2}\right) \stackrel{\text{VLSF}}{=} \lim_{x \rightarrow 0} \sin x = 0$$

$$\frac{n}{2+n^2} > 0 \quad \left(\text{L'Hôpital's rule}\right) \left(\frac{x}{2+x^2}\right)' = \frac{2+x^2 - x \cdot 2x}{(2+x^2)^2} = \frac{2-x^2}{(2+x^2)^2} < 0$$

per $x \geq 2$ (hier per $n \geq 2$)

\Rightarrow Monotonie $\frac{n}{2+n^2}$ a. bes. f. hat $a_n = \frac{n}{2+n^2}$ ist abnehmend

⇒ (Leibniz's rule) $\sum (-1)^k \binom{n}{k} \left(\frac{a}{1+n}\right)^k$ & 'L'Hôpital',
 def, dove' vado sempre' a calcolare'

3)

$$f(x) = \sqrt{\arctan(x-1)^2}$$

a) $\arctan(x-1)^2 \geq 0 \Rightarrow f(x)$ def. v \mathbb{R} , $f(x)$ è ppa' e derivabile,

$$b) f'(x) = \frac{1}{2\sqrt{\arctan(x-1)^2}} \cdot \frac{1}{1+(x-1)^4} \cdot 2(x-1) \text{ per } x \neq 1$$

$$f'_{\pm}(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x-1} = \lim_{x \rightarrow 1 \pm} \frac{\sqrt{\arctan(x-1)^2}}{x-1} = \frac{0}{0}$$

$$= \lim_{x \rightarrow 1 \pm} \sqrt{\frac{\arctan(x-1)^2}{(x-1)^2}} \cdot \operatorname{sgn}(x-1) = \begin{cases} +1 \\ -1 \end{cases} \Rightarrow$$

$$\lim_{t \rightarrow 0} \frac{\arctan t}{t} = 1 \quad (\text{VLSF})$$

f non è' in $x=1$ derivabile' attraverso, già' è' derivabile',
 $f'_+(1) = 1, f'_-(1) = -1$

$$\text{altro: } \lim_{x \rightarrow 1+} \frac{\sqrt{\arctan(x-1)^2}}{x-1} = \lim_{x \rightarrow 1+} \frac{\sqrt{\arctan(x-1)^2}}{\sqrt{(x-1)^2}} = 1$$

$$\lim_{x \rightarrow 1-} \frac{\sqrt{\arctan(x-1)^2}}{x-1} = \lim_{x \rightarrow 1-} \frac{\sqrt{\arctan(x-1)^2}}{-\sqrt{(x-1)^2}} = \lim_{x \rightarrow 1-} -\frac{\sqrt{\arctan(x-1)^2}}{\sqrt{(x-1)^2}} = -1$$

Ma' non è' (già' derivabile) derivabile' attraverso

$$f'_+(1) = \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{\arctan(x-1)^2}} \cdot \frac{1}{1+(x-1)^4}$$

"stessa" limite
 "già' è' derivabile"

(4) $f(x) = |x+2| e^{\frac{1}{x}}$

a) $D_f = \mathbb{R} \setminus \{0\}$; $f(x) \geq 0$, $f(x) = 0 \Leftrightarrow x = -2$ - global minimum free

b) $\lim_{x \rightarrow \pm\infty} |x+2| e^{\frac{1}{x}} = " \infty \cdot 1 " = +\infty \Rightarrow f$ nemá globalného maxima

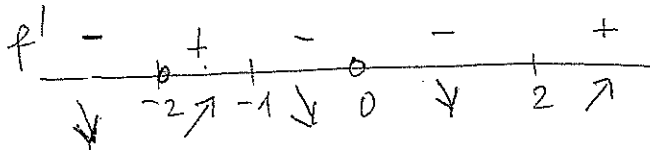
c) lokálne extrém:

$f(x) = (x+2) e^{\frac{1}{x}} \cdot \text{sgn}(x+2)$ v $\mathbb{R} \setminus \{0\}$, $x \neq -2$

$f'(x) = \left(e^{\frac{1}{x}} + (x+2) e^{\frac{1}{x}} \left(-\frac{1}{x^2} \right) \right) \text{sgn}(x+2) =$

$= e^{\frac{1}{x}} \text{sgn}(x+2) \left(1 - \frac{x+2}{x^2} \right) = e^{\frac{1}{x}} \text{sgn}(x+2) \frac{x^2 - x - 2}{x^2} =$

$f'(x) = 0 \Leftrightarrow x^2 - x - 2 = 0 \Leftrightarrow (x-2)(x+1) = 0 \Leftrightarrow \begin{cases} x = -1 \vee x = 2 \\ \frac{e^{\frac{1}{x}}}{x^2} \cdot \text{sgn}(x+2) \end{cases}$



v $x = -1$ f má lok. maximum,

$f(-1) = \frac{1}{e}$

v $x = 2$ f má lok. minimum,

$f(2) = 4\sqrt{e}$ - nové glob. min.

graf:

limita f v $x=0$ a D_f $\lim_{x \rightarrow 0^-} |x+2| e^{\frac{1}{x}} = 0$, $\lim_{x \rightarrow 0^+} |x+2| e^{\frac{1}{x}} = +\infty$

$f'_+(2) = \lim_{x \rightarrow 2^+} f'(x) = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

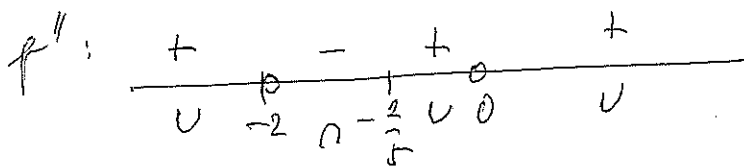
$f'_-(-2) = -e^{-\frac{1}{2}} = -\frac{1}{\sqrt{e}}$

Wichtige grafen:

(Nullstellen - Extrema, alle xi fce Extrema, resp. Wendepunkte, inflexion' brdf)

$$\begin{aligned}
 f''(x) &= \left(e^{\frac{1}{x}} \left(1 - \frac{1}{x} - \frac{1}{x^2} \right) \operatorname{sgn}(x+2) \right)' \quad x \neq -2 \\
 &= \operatorname{sgn}(x+2) e^{\frac{1}{x}} \left[-\frac{1}{x^2} \left(1 - \frac{1}{x} - \frac{2}{x^2} \right) + \left(\frac{1}{x} + \frac{4}{x^3} \right) \right] = \\
 &= \operatorname{sgn}(x+2) \cdot e^{\frac{1}{x}} \cdot \frac{1}{x^4} (-x^2 + x + 2 + x^2 + 4x) = \\
 &= \operatorname{sgn}(x+2) e^{\frac{1}{x}} \cdot \frac{1}{x^4} (5x + 2) \quad , \quad x \neq -2
 \end{aligned}$$

$f''(x) = 0 \Leftrightarrow x = -\frac{2}{5}$



$x = -\frac{2}{5}$ ist inflexion (mon'e, $\lim_{x \rightarrow 0} f'(x) = 0$)