MA1 – DÚ 1

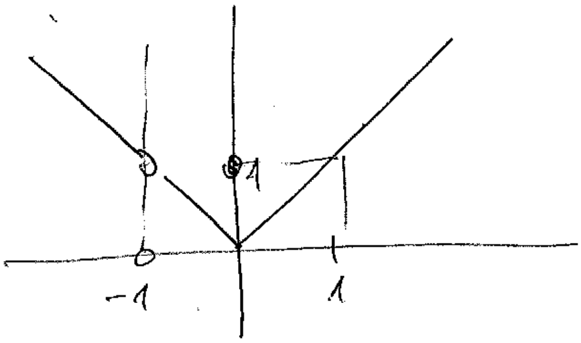
1

$$f(x)=\sqrt{x^2-\frac{2x^3+3x^2-1}{x^2+2x+1}}=\sqrt{\frac{x^4+2x^3+x^2-2x^3-3x^2+1}{(x+1)^2}}=$$

$$=\sqrt{\frac{x^4-2x^2+1}{(x+1)^2}}=\sqrt{\frac{(x^2-1)^2}{(x+1)^2}}=\sqrt{(x-1)^2}=|x-1|$$

$D_f = R \setminus \{-1\} = (-\infty,-1) \cup (-1,+\infty)$

graf:



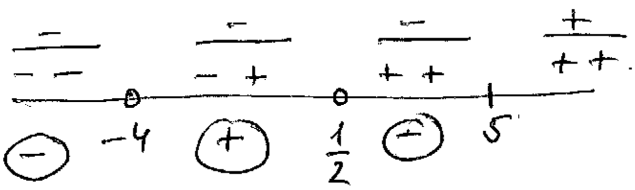
2

$$\frac{1}{2x-1} \geqq \frac{1}{x+4} \iff \frac{1}{2x-1}-\frac{1}{x+4} \geqq 0 \qquad x \neq \tfrac{1}{2}, -4$$

$$\frac{x+4-2x+1}{(2x-1)(x+4)} \geqq 0$$

$$\frac{-x+5}{(2x-1)(x+4)} \geqq 0 \iff$$

$$\iff \frac{x-5}{(2x-1)(x+4)} \leq 0$$



$x \in (-\infty,-4) \cup (\tfrac{1}{2}, 5\rangle$

3

$|x+1| \leqq 2 \ \wedge \ |x-1| \geqq 3$

$x \in \langle -3,1 \rangle \ \wedge \ x \in (-\infty,-2\rangle \cup \langle 4,+\infty) \iff$

$\iff x \in \langle -3,1\rangle \cap \left((-\infty,-2\rangle \cup \langle 4,+\infty) \right) \iff$

$x \in \langle -3,-2 \rangle$

(4) v $\langle 0, 2\pi)$: $2\cot g^2 x = \dfrac{3}{\sin x}$ $\qquad x \neq 0, \pi$

$$2\cos^2 x = 3\sin x$$

$$2(1-\sin^2 x) - 3\sin x = 0$$

$$-2\sin^2 x - 3\sin x + 2 = 0$$

$$2\sin^2 x + 3\sin x - 2 = 0 \qquad \text{subst. } \sin x = y$$

$$2y^2 + 3y - 2 = 0$$

$$y_{1,2} = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4} = \begin{cases} \frac{1}{2} \\ -2 \end{cases}$$

tj. dostaneme : $\sin x = \frac{1}{2}$ $\quad (|\sin x| \leq 1)$

$$x_1 = \frac{\pi}{6}, \quad x_2 = \pi - \frac{\pi}{6} = \frac{5}{6}\pi$$

(5) $\dfrac{1}{\log x} \geqq \log x$ $\qquad$ (log – dekadický logaritmus)

1) $x > 0$, $x \neq 1$

2) $\dfrac{1}{\log x} - \log x \geqq 0$

$$\frac{1 - \log^2 x}{\log x} \geqq 0 \quad (\text{subst. } \log x = y) : \quad \frac{1-y^2}{y} \geqq 0 \iff$$

$$\iff y \in (-\infty, -1\rangle \cup (0, 1\rangle$$

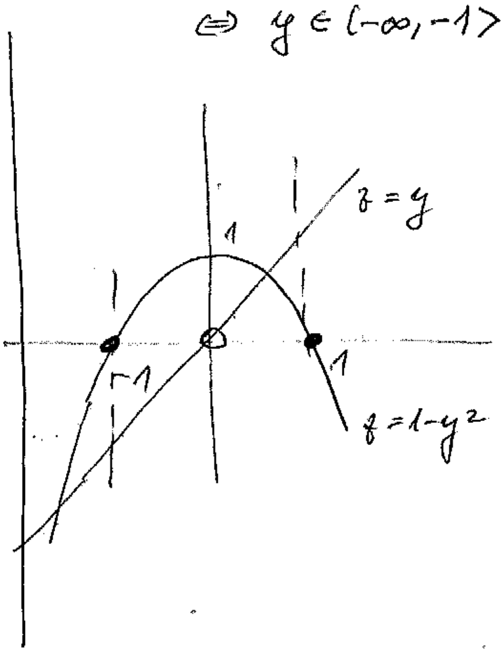
tj. $\log x \leq -1$ $\vee$

$\vee$ $0 < \log x \leq 1$

$$\iff x \in \left(0, \tfrac{1}{10}\right\rangle \cup (1, 10\rangle$$

($\log x \leq -1$ $\quad \vee \quad$ $0 < \log x \leq 1$

$0 < x \leq 10^{-1}$ $\qquad$ $1 < x \leq 10$)



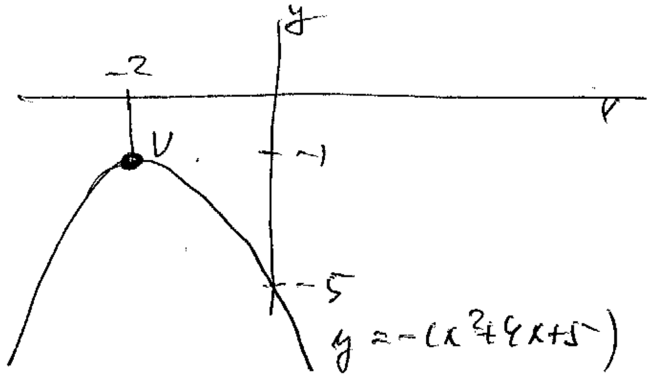
(6) *grafy:*

$f(x) = -(x^2+4x+5)$

$= -\left[(x+2)^2+1\right]$

$= -(x+2)^2 - 1$

$V[-2,-1]$



$g(x) = -1 + \sqrt{x+4}$

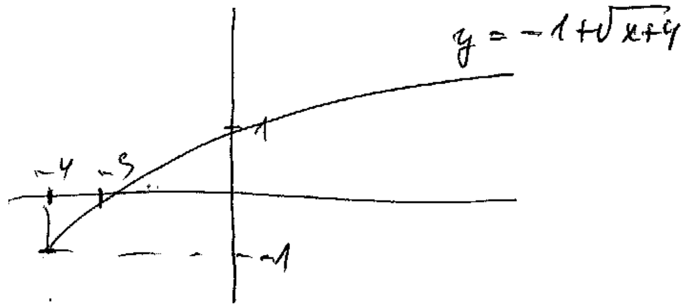
$x \geq -4$

$g(-4) = -1, \quad g(0) = 1$

$g(x) = 0 \iff \sqrt{x+4} = 1$

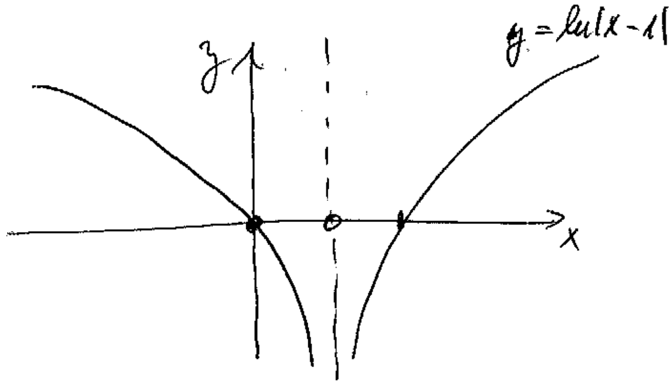
$x+4 = 1$

$x = -3$



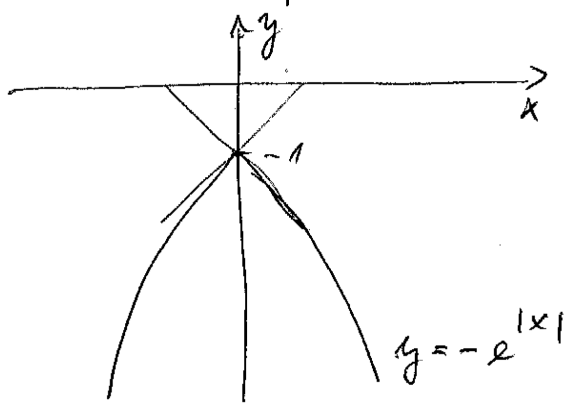
$h(x) = \ln|x-1|$

$\mathcal{D}_h = R - \{1\}, \; h(0)=0, \; h(2)=0$



$k(x) = -e^{|x|}$

$\mathcal{D}_f = R, \quad k(0) = -1, \quad k(x) < 0 \text{ v } R$



(7) $f(x) = \dfrac{x+1}{x-2}, \quad \mathcal{D}_f = (-\infty, 2) \cup (2, +\infty)$

inverzní fce: $f(x) = y \iff x = f^{-1}(y)$?

$f(0) = -\frac{1}{2}, \quad f(x) = 0 \iff x = -1$

g. $\dfrac{x+1}{x-2} = y$

$x+1 = y(x-2)$

$x(1-y) = -2y-1$ per $y \neq 1$

$x = \dfrac{2y+1}{y-1}$ (=

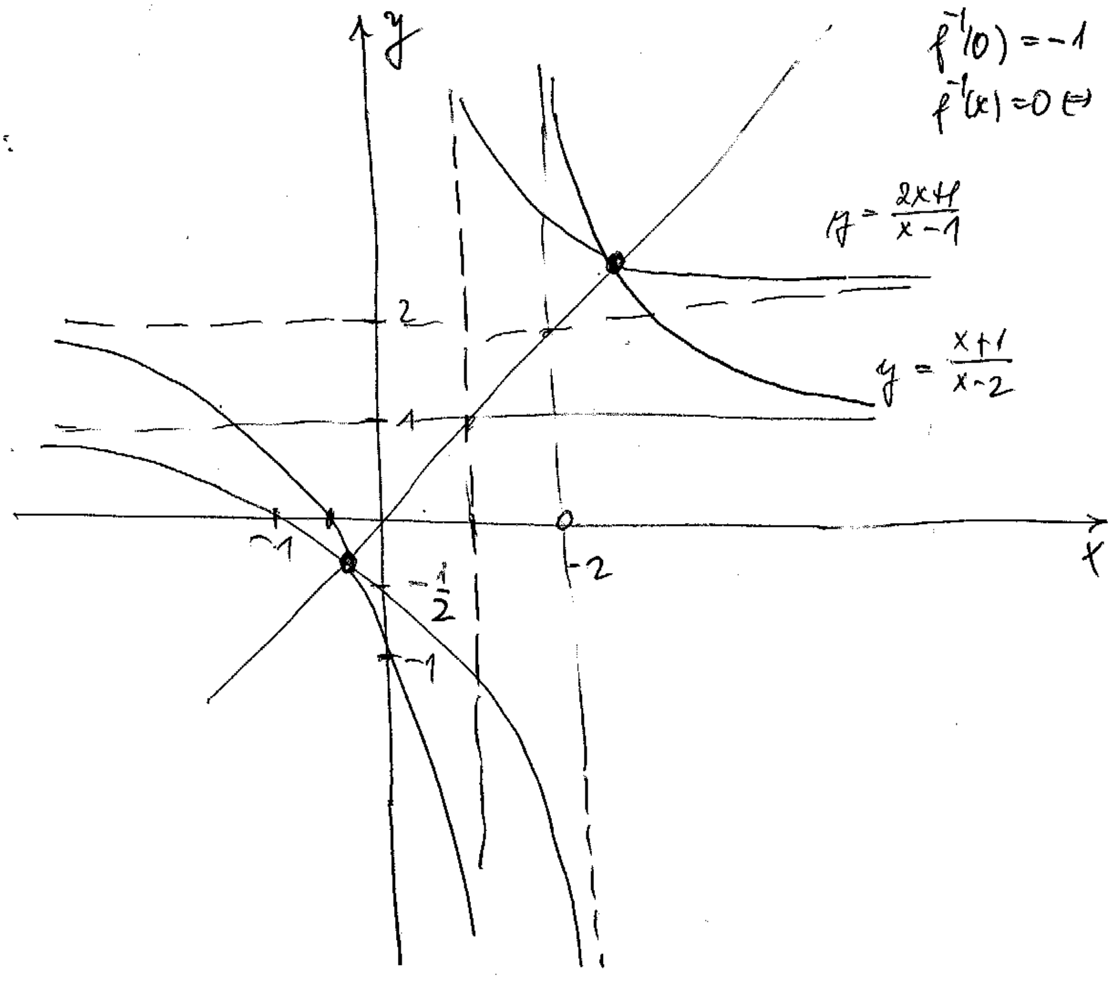
$x \leftrightarrow y$: $y = \dfrac{2x+1}{x-1}$, $x \in (-\infty, 1) \cup (1, +\infty)$

h. $f^{-1}(x) = \dfrac{2x+1}{x-1}$ – asymptoty $x = 1$, $y = 2$

$f^{-1}(0) = -1$

$f^{-1}(x) = 0 \Leftrightarrow x = -\frac{1}{2}$

graf:



$$f^{-1}(x) = \frac{2x-2+3}{x-1} = 2 + \frac{3}{x-1}$$